B. Tech Degree IV Semester Examination, April 2008

IT/CS/EC/CE/ME/SE/EB/EI/EE 401
ENGINEERING MATHEMATICS III
(2006 Scheme)

Time : 3 Hours — Maximum Marks : 100

PART - A
(Answer ***ALL*** questions)

(8 x 5 = 40)

I. (a) An electric field in the xy – plane is given by the potential function $\phi = 3x^2y - y^3$. Find the stream function.

(b) Find the bilinear transformation which maps the points $z = 1, i^o, -1$ into the points $w = 0, 1, \infty$.

(c) Evaluate $\int_c \frac{e^{2z}}{(z+1)^4} dz$ where c is the circle $|z| = 2$.

(d) Evaluate $\int_c \tan z dz$ where c is the circle $|z| = 2$.

(e) Find the differential equation of all spheres whose centres lie in the xy – plane.

(f) Solve $z^2\left(px^2 + q^2\right) = 1$.

(g) Find the solution of one dimensional heat equation.

(h) A string is stretched and fastened to two points ℓ apart. Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{\ell}\right)$ from which it is released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by

$$y(x,t) = a\sin\left(\frac{\pi x}{\ell}\right)\cos\left(\frac{\pi ct}{\ell}\right).$$

PART - B

II. (a) If $F(z)$ is an analytic function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|RF(z)|^2 = 2|F'(z)|^2$. (6)

(b) Determine the region of the w – plane into which the region $\frac{1}{2} \le x \le 1$ and $\frac{1}{2} \le y \le 1$ mapped by the transformation $w = z^2$. (9)

OR

III. (a) Determine the analytic function $F(z)$ in terms of z whose real part is $e^{2x}\left(x\cos 2y - y\sin 2y\right)$. (6)

(Turn Over)

(b) Show that for $F(z)=\dfrac{2xy(x+iy)}{x^2+y^2}$ *if* $z=0$

$=0$ *if* $z=0$

satisfies the C.R equations at the origin but $F'(z)$ does not exist at origin. (9)

IV. (a) Expand $\dfrac{1}{z^2-3z+2}$ in the region

(i) $1<|z|<2$ (ii) $|z|>2$ (iii) $0<|z-1|<1$. (8)

(b) Prove that $\int_{-\infty}^{\infty}\dfrac{\cos mx}{\left(x^2+a^2\right)(x^2+b^2)}dx=\dfrac{\pi}{a^2-b^2}\left(\dfrac{e^{-bm}}{b}-\dfrac{e^{-am}}{a}\right)$. (7)

OR

V. (a) Verify Cauchy's theorem if $F(z)=z+1$ and C is the boundary of the square whose vertices are at the points $z=0, z=1, z=1+i^o, z=i^o$. (8)

(b) Evaluate $\int_{o}^{2\pi}\dfrac{d\theta}{1-2r\cos\theta+r^2}=\dfrac{2\pi}{1-r^2}(0<r<1)$. (7)

VI. (a) Form the partial differential equation by eliminating the arbitrary functions from

$$F\left(x+y+z, x^2+y^2+z^2\right)=0$$ (8)

(b) Solve $p^2+q^2=z^2(x+y)$. (7)

OR

VII. (a) Solve $x^2(y-z)p+y^2(z-x)q-z^2(x-y)=0$. (7)

(b) Solve $\dfrac{\partial^2 z}{\partial x^2}+\dfrac{\partial^2 z}{\partial x\partial y}-6\dfrac{\partial^2 z}{\partial y^2}=\cos(2x+y)+5e^{3x+y}$. (8)

VIII. The ends A and B of a rod 20 cm long have the temperature at 30°C and 80°C until steady state prevails. The temperature of the ends are changed into 40°C and 60°C respectively. Find the temperature distribution in the rod at time *t*. (15)

OR

IX. A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge $y=0$ is given by

$u=20x$ for $0\le x\le 5$

$=20(10-x)$ for $5\le x\le 10$

and the two long edges $x=10, x=0$ as well as the other short edge are kept at 0°C. Prove that the temperature u at any point is given by

$$u=\frac{800}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{(2n-1)^2}\sin\frac{(2n-1)\pi x}{10}\cdot e^{\frac{-(2n-1)\pi y}{10}}.$$ (15)

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