

B. Tech Degree III Semester Examination, November 2009**IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 301 ENGINEERING MATHEMATICS II**
(2006 Scheme)

Time : 3 Hours

Maximum Marks : 100

PART – A
(Answer all questions)

(8 x 5=40)

- I. a. Write the vector $V=(2, -5, 3)$ in R^3 as a linear combination of the vectors $e_1=(1, -3, 2)$, $e_2=(2, -4, -1)$ and $e_3=(1, -5, 7)$, if possible.

- b. Solve the following system of equation by matrix inversion method:

$$3x + y + 2z = 3$$

$$2x - 3y - z = -3$$

$$x + 2y + z = 4$$

- c. Expand the function defined by

$$f(x) = 0 \text{ for } -2 < x < 0$$

$$= x \text{ for } 0 < x < 2$$

as a full series in $[-2, 2]$.

- d. Find the Fourier sine transform of $e^{-|x|}$.

- e. Find the Laplace transform of $\frac{1 - \cos 2t}{t}$.

- f. Evaluate $\int_0^{\infty} t e^{-2t} \sin t \, dt$ using Laplace transforms..

- g. Prove that $\nabla^2 (r^n \bar{r}) = n(n+3)r^{n-2}\bar{r}$.

- h. Obtain a, b, c such that the vector $F = (x + y + az)i + (bx + 2y - z)j + (-x + cy + 2z)k$ is irrotational.

PART B

(4 x 15 = 60)

- II. a. Show that the equations $x + y + z = a$, $3x + 4y + 5z = b$, $2x + 3y + 4z = c$

(i) have no solution if $a=b=c=1$

(ii) have many solutions if $a = \frac{b}{2} = c = 1$. (8)

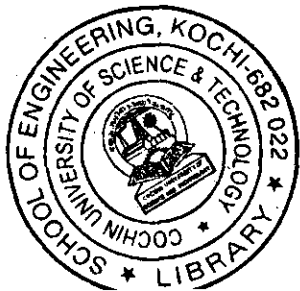
- b. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation. (7)

OR

- III. a. Determine the values of 'k' for which the following set of equation may possess non-trivial solution and find it.

$$3x_1 + x_2 - kx_3 = 0, 4x_1 - 2x_2 - 3x_3 = 0, 2kx_1 + 4x_2 + kx_3 = 0. \quad (8)$$

(Turn over)



- b. Check whether the set $w = \{(a, b, 2a + 3b) : ab \in R\}$ is a subspace or not. (4)
 c. Examine whether $(1, 0, 1), (1, 1, 0), (0, 1, 1)$ is a basis of R^3 . (3)

- IV. a. Find the Fourier series to represent $f(x)$ in $[-\pi, \pi]$, given

$$f(x) = \begin{cases} x + x^2, & -\pi < x < \pi \\ \pi^2, & x = \pm \pi \end{cases}$$

Deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$. (8)

- b. Solve the integral equation: $\int_0^{\infty} f(x) \cos \alpha x dx = e^{-\alpha}$. (7)

OR

- V. a. Find the Fourier cosine transform of e^{-x^2} . (10)

- b. If the Fourier sine transform of $f(x)$ is $\frac{1 - \cos x\pi}{x^2 \pi^2}$, $0 < x < \pi$, find $f(x)$. (5)

- VI. a. Prove that $\int_0^{\infty} \frac{e^{-t} \sin^2 t}{t} dt = \frac{1}{4} \log 5$. (8)

- b. Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{5}\right)$. (7)

OR

- VII. a. Use Laplace transform method to solve $y'' - 3y' + 2y = 4t + e^{3t}$ when $y(0) = 1, y'(0) = -1$. (8)

- b. Find the Laplace transform of $e^{-3t} u(t - 2)$. (7)

- VIII. a. Using Divergence theorem, calculate $\iiint \vec{F} \cdot \vec{x} ds$ where $\vec{F} = 2xz i + yz j + z^2 k$ over the half sphere $x^2 + y^2 + z^2 = a^2$ lying above the IC plane. (6)

- b. Verify Stokes theorem for the function $\vec{f} = (x^2 + y^2)i - 2xy j$ around the rectangle bounded by the lines $x = \pm a, y = 0, y = b$. (9)

OR

- IX. a. Find the directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ at $(2, -1, 1)$ in the direction of the vector $i + 2j + 2k$. (5)

- b. Evaluate $\oint_C xy dx + xy^2 dy$ by Stoke's theorem where c is the square in the xy plane with vertices $(1, 0), (-1, 0), (0, 1), (0, -1)$. (10)
