BTS(C) – III – 08 – 062 - B

B.Tech. Degree III Semester Examination, November 2008

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 301 ENGINEERING MATHEMATICS II

(2006 Scheme)

Time: 3 Hours — Maximum Marks: 100

(8 x 5 = 40)

PART A

(Answer All questions)

I a) Define the rank of a matrix. Find the rank of A by reducing to the normal form.

$$A\begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$$

b) Let $V = \{f / f$ is a real valued continuous function on $[a,b]$ such that $f(b) = p, p \neq 0\}$. Then verify whether V is a vector space.

c) Express $f(x) = |x|, -\pi < x < \pi$ as a Fourier series.

d) Find the Fourier transform of the function $f(t) = e^{-a|t|}, -\alpha < t < \alpha, a > 0$.

e) Find the Laplace transform of :

i) $u_a(t)$, unit step function

ii) $f(t) = t^2 u_3(t)$

f) Using Laplace transform, solve the initial value problem $y'' + 3y' + 2y = 3$, given $y(0) = 1, y'(0) = 1$.

g) If $\bar{r} = xi + yj + zk$, and $r = |\bar{r}|$, then prove that $\nabla r^n = nr^{n-2}\bar{r}$.

h) If $\bar{v} = 3x^2y^2z^4 i + 2x^3yz^4 j + 4x^3y^2z^3 k$, show that $\bar{v}$ is a conservative field and find its scalar potential.

PART B

(4 x15 = 60)

II a) If $A\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that $A^n = A^{n-2} + A^2 - I, n \geq 3$. Hence find A^{50}. (10)

b) Find the dimension of the subspace of R^4 spanned by $\{(1,0,0,0), (0,1,0,0), (1,2,0,1), (0,0,0,1)\}$. Hence find its basis. (5)

OR

III a) Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (8)

b) Let T be a linear transformation from R^2 to R^3 such that $Tx = Ax$, where $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 3 & 2 \end{bmatrix}$. Find ker (T), ran (T) and their dimensions. (7)

(Turn Over)

IV a) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots\dots\dots\dots = \frac{\pi^2}{12}$. (10)

b) Find the Fourier transform of the function $f(t) = e^{-at^2}, a > 0$. (5)

OR

V a) Find the Fourier cosine and sine integral representation of $f(x) = e^{-kx}, x \geq 0$, where k is a positive constant. (10)

b) Using convolution find the inverse Fourier transform of $\frac{1}{12 + 7iw - w^2}$. (5)

VI a) Find the Laplace transform of

i) $(Cot + Sin\, t)^2$ ii) $e^t Sin\, t$ (7)

b) Find the inverse Laplace transform of

i) $\frac{5S^2 + 3S - 16}{(S-1)(S-2)(S+3)}$ ii) $\frac{3}{S^2 + 2S}$ (8)

OR

VII a) Using convolution find the inverse Laplace transform of $\frac{1}{(S^2 + \omega^2)^2}$. (7)

b) Find the Laplace transform of the periodic function defined by the triangular wave,

$$f(t) = \frac{t}{a}, 0 \leq t \leq a$$

$$\frac{za - t}{a}, a \leq t \leq 2a, f(t + 2a) = f(t)$$ (8)

VIII a) Prove that Curl$\left(f\bar{v}\right) = (grad\ f) \times \bar{v} + f(Curl\ \bar{v})$, where f is a scalar function and $\bar{v}$ is a vector function. (10)

b) Find the directional derivative of the scalar function $f = x^2y - y^2z - xyz$ at $(1, -1, 0)$ in the direction of $i - j + 2k$. (5)

OR

IX a) Evaluate the surface integral $\iint_S \bar{F}\, \hat{n}\, d\, A$, where $\bar{F} = 6zi + 6j + 3yk$ and S is the portion of the plane $2x + 3y + 4z = 12$ which is in the first octant. (8)

b) Let D be the region bounded by the closed cylinder $x^2 + y^2 = 16, z = 0$ and $z = 4$. Verify the divergence theorem if $\bar{v} = 3x^2 i + 6y^2 j + zk$. (7)

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